

Evaluate $\int_0^5 \frac{12-3z}{z^2-8z+12} dz$.

$$z^2 - 8z + 12 = (z-2)(z-6)$$

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$$= \int_0^2 \frac{12-3z}{z^2-8z+12} dz + \int_2^5 \frac{12-3z}{z^2-8z+12} dz$$

2

$$= \lim_{N \rightarrow 2^-} \int_0^N \frac{12-3z}{z^2-8z+12} dz$$

$$= \lim_{N \rightarrow 2^-} \left(-\frac{3}{2} \ln|z^2-8z+12| \right) \Big|_0^N$$

$$= \lim_{N \rightarrow 2^-} -\frac{3}{2} (\ln|N^2-8N+12| - \ln|2|)$$

$$= -\frac{3}{2} (-\infty - \ln|2|) \rightarrow \infty$$

$$\int \frac{12-3z}{z^2-8z+12} dz$$

$$= -\frac{3}{2} \int \frac{1}{u} du$$

$$u = z^2 - 8z + 12$$

$$du = (2z - 8) dz$$

$$= -\frac{3}{2} \ln|u|$$

$$-\frac{3}{2} du = (12-3z) dz$$

$$= -\frac{3}{2} \ln|z^2-8z+12|$$

INTEGRAL DIVERGES

Evaluate $\int \frac{5x^2 - 20}{x^3 - 4x^2 + 20x} dx$.

$$x^3 - 4x^2 + 20x = x(x^2 - 4x + 20)$$

$$= x((x-2)^2 + 16)$$

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$$= \int \left(\frac{A}{x} + \frac{B(2x-4)+C}{(x-2)^2+16} \right) dx = \int \left(\frac{1}{x} + \frac{3(2x-4)+8}{(x-2)^2+16} \right) dx$$

$$5x^2 - 20 = A[(x-2)^2 + 16] + B(2x-4)x + Cx$$

$x=0$: $-20 = 20A \rightarrow A = -1$

$x=2$: $0 = 16A + 2C \rightarrow C = -8A = 8$

COEF OF x^2 : $5 = A + 2B \rightarrow B = \frac{1}{2}(5 - A) = 3$

SANITY CHECK:

$$x=3 \quad \frac{45-20}{27-36+60} \stackrel{?}{=} -\frac{1}{3} + \frac{3(2)+8}{17}$$

$$\frac{25}{51} \stackrel{?}{=} -\frac{1}{3} + \frac{14}{17} = \frac{-17+42}{51} \checkmark$$

$$= -\ln|x|$$

$$+ 3 \ln|x^2 - 4x + 20|$$

$$+ \frac{8}{4} \tan^{-1} \frac{x-2}{4} + C$$

$$\frac{1}{2}$$

$$= -\ln|x|$$

$$\frac{1}{2}$$

$$+ 3 \ln|x^2 - 4x + 20|$$

$$1$$

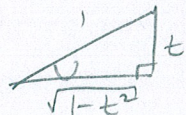
$$+ 2 \tan^{-1} \frac{x-2}{4} + C$$

Evaluate $\int e^{\arcsin t} dt$.

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EITHER VERSION OK

①
$$\begin{cases} u = \arcsin t \\ t = \sin u \\ dt = \cos u du \end{cases}$$



	u	dv
$\cos u$	$+$	e^u
$-\sin u$	$-$	e^u
$-\cos u$	$+$	e^u

①
$$\int e^u \cos u du = e^u \cos u + e^u \sin u - \int e^u \cos u du \quad \text{③}$$

$$2 \int e^u \cos u du = e^u \cos u + e^u \sin u$$

$$\int e^u \cos u du = \frac{1}{2} e^u \cos u + \frac{1}{2} e^u \sin u + C \quad \text{①}$$

$$= \frac{1}{2} e^{\arcsin t} \sqrt{1-t^2} + \frac{1}{2} t e^{\arcsin t} + C$$

①

①

Evaluate $\int_0^{\infty} y^2 e^{-\frac{y}{2}} dy$.

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$$\textcircled{1} = \lim_{N \rightarrow \infty} \int_0^N y^2 e^{-\frac{y}{2}} dy$$

$$\textcircled{1} = \lim_{N \rightarrow \infty} \left[-(2y^2 + 8y + 16)e^{-\frac{y}{2}} \right]_0^N$$

$$\textcircled{1} = \lim_{N \rightarrow \infty} \left(-(2N^2 + 8N + 16)e^{-\frac{N}{2}} + 16 \right)$$

$$= -0 + 16$$

$$= \boxed{16}$$

$\frac{1}{2}$

$$\lim_{N \rightarrow \infty} \frac{2N^2 + 8N + 16}{e^{\frac{N}{2}}} \frac{\infty}{\infty}$$

$$= \lim_{N \rightarrow \infty} \frac{4N + 8}{\frac{1}{2}e^{\frac{N}{2}}} \frac{\infty}{\infty}$$

$$= \lim_{N \rightarrow \infty} \frac{4}{\frac{1}{4}e^{\frac{N}{2}}} \frac{4}{\infty}$$

$$= \boxed{0}$$

$\frac{1}{2}$

$$\begin{array}{r} u \quad dv \\ y^2 \quad e^{-\frac{y}{2}} \\ 2y \quad -2e^{-\frac{y}{2}} \\ 2 \quad 4e^{-\frac{y}{2}} \\ 0 \quad -8e^{-\frac{y}{2}} \end{array}$$

$$\int y^2 e^{-\frac{y}{2}} dy$$

$$= -(2y^2 + 8y + 16)e^{-\frac{y}{2}}$$